Physics III
ISI B.Math
Mid Semestral Exam : September 7, 2009

Total Marks: 100
Answer any five questions

1. (i) One of the following two fields is an impossible electrostatic field. Which one? Justify your answer.
(a) $\mathbf{E} = k[xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}]$
(b) $\mathbf{E} = k[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}]$ (6)

(ii) Evaluate the integral

$$J = \int_V e^{-r}\left(\nabla \cdot \frac{\hat{\mathbf{r}}}{r^2}\right) d\tau$$

where V is a sphere of radius R , centred at the origin. (6)
(iii) The sphere of radius a was filled with positive charge at uniform density

ρ. Then a smaller sphere of radius $\frac{a}{2}$ was carved out as shown in the figure, and left empty. What is the direction and magnitude of the electric field at A? at B?(8)

2. A static charge distribution produces a radial electric field

$$\mathbf{E} = A\frac{e^{-br}}{r}\hat{\mathbf{r}}$$

where A and b are constants.
(a) What is the charge density ? Sketch it.(12)
(b) What is the total charge Q?(8)

3. (a) Justify **Earnshaw's Theorem**: *A charged particle cannot be held in a stable equilibrium by electrostatic forces alone.*(5)
(b) Two uniform infinite sheets of electric charge densities $+\sigma$ and $-\sigma$ intersect at right angles. Find the magnitude and direction of the electric field everywhere and sketch the lines of $\mathbf{E}$.(8)
(c) Find the total energy of a uniformly charged spherical shell of total charge q and radius R.(7)

4. Two infinite grounded metal plates lie parallel to the xz plane, one at $y = 0$, the other at $y = a$. The left end, at $x = 0$, is closed off with an infinite strip insulated from the two plates and maintained at a constant potential V_0. Find the potential inside this "slot".(20)

5. Charges $+q$, $-q$ lie at the points $(x, y, z) = (a, 0, a), (-a, 0, a)$ above a grounded conducting plane at $z = 0$. Find
(a) The total force on the charge $+q$.(6)
(b) The work done against the electrostatic forces in assembling this system of charges.(6)
(c) The surface charge density at the point $(a, 0, 0)$.(8)

6. (a) Find the potential $V(r, \theta)$ and the corresponding electric field for an electric dipole of dipole moment $\mathbf{p}$. Choose the dipole to be at the origin with its dipole moment pointing in the z direction.(10)
(b) Find the bound charges and the electric field produced by a uniformly polarized sphere (with polarization $\mathbf{P}$) of radius R. [*Hint: model the uniformly polarized sphere as two uniformly charged spheres one positively charged and one negatively charged superposed with a slight displacement of their centres. Justify this model in a few lines before you proceed.*](10)